WNE Linear Algebra Final Exam

Series B

10 March 2016

Please use separate sheets for different problems. Please provide the following data on each sheet

- **name, surname and your student number,**
- **number of your group,**
- **number of the corresponding problem and its series.**

Problem 1.
Let $V = \text{lin}((1,-2,2,1,0),(0,0,1,0,1),(2,-4,5,2,1),(-1,2,-1,-1,1))$ be a subspace of $\mathbb{R}^5$.

a) find dimension of V and a system of linear equations which set of solutions is equal to V,

b) let $w = (-1,2,1,t,3) \in \mathbb{R}^5$. For which $t \in \mathbb{R}$ the subspace $\text{lin}(w)$ is a subset of V, i.e. $\text{lin}(w) \subset V$?

Problem 2.
Let $\alpha_1 = (0,1,2), \alpha_2 = (0,1,3), \alpha_3 = (1,0,1)$ be three vectors in $\mathbb{R}^3$.

a) which of the sequences below are ordered bases of $\mathbb{R}^3$? give a short explanation in each case
   i) $(\alpha_1, \alpha_1 - \alpha_2, \alpha_1 - \alpha_3, \alpha_3)$,
   ii) $(\alpha_1, -\alpha_2, \alpha_1 + \alpha_3)$,
   iii) $(\alpha_2, 2\alpha_3)$.

b) find coordinates of the vector $-\alpha_1+\alpha_2+\alpha_3$ relative to the ordered basis $(\alpha_1, \alpha_2+\alpha_3, \alpha_3)$.

Problem 3.
Let $A = \begin{bmatrix} s & 1 & 1 \\ 0 & 2 & 3 \\ 0 & 1 & 0 \end{bmatrix} \in M(3 \times 3; \mathbb{R})$.

a) check if matrix A is diagonalizable for $s = 3$,

b) for $s = 2$ find matrix $C \in M(3 \times 3; \mathbb{R})$ such that $C^{-1}AC = \begin{bmatrix} a & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & b \end{bmatrix}$ for some $a, b \in \mathbb{R}$.

Problem 4.
Let $\mathcal{A} = ((1,0,1),(0,1,0),(1,0,2))$ be an ordered basis of $\mathbb{R}^3$ and let $\mathcal{B} = ((0,1),(1,-1))$ and $\mathcal{C}$ be ordered bases of $\mathbb{R}^2$. The linear transformation $\varphi\colon \mathbb{R}^3 \longrightarrow \mathbb{R}^2$ is given by the matrix $M(\varphi)_{\mathcal{A}}^{\mathcal{B}} = \begin{bmatrix} -1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$.

The basis $\mathcal{C}$ of $\mathbb{R}^2$ is given by the matrix $M(id)_{\mathcal{C}}^{\mathcal{B}} = \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix}$.

a) compute basis $\mathcal{C}$,
b) find formula of φ.

Problem 5.
Let $V = \operatorname{lin}((1,0,-1),(1,1,0),(3,1,-2))$ be a subspace of $\mathbb{R}^3$.
a) find an orthonormal basis of $V^{\perp}$,
b) compute the orthogonal projection of $w = (0,0,1)$ onto V.

Problem 6.
Let
$$A = \begin{bmatrix} 1 & 0 & 3 \\ 1 & 0 & r \\ 1 & 1 & 1 \end{bmatrix}.$$
a) for which $r \in \mathbb{R}$ the matrix A is invertible?,
b) for which $r \in \mathbb{R}$ the entry in the second row and the second column of the matrix A^{-1} is equal to 2?

Problem 7.
The affine space $H \subset \mathbb{R}^3$ is given by the equation $x_1 + x_2 - x_3 = 1$.
a) find a parametrization of the line L perpendicular to H and passing through $P = (1,1,0)$,
b) find the orthogonal projection of P onto H.

Problem 8.a) bring the following linear programming problem to a standard form
$x_2 + x_3 \longrightarrow max$
$$\begin{cases} -x_1 + 2x_2 - x_3 \geqslant 2 \\ x_3 \leqslant 1 \\ x_1, x_2 \geqslant 0 \end{cases}$$
b) solve the following linear programming problem using simplex method
$x_1 + 2x_2 \longrightarrow min$
$$\begin{cases} x_1 + x_2 = 1 \\ -2x_1 + x_3 = 0 \\ 2x_1 + x_4 = 4 \end{cases} \text{ and } x_i \geqslant 0 \text{ for } i = 1, \dots, 4$$